Abstract

This work aims to study the *stability* of chaotic dynamical systems starting from the geometrical structure of their *attractors* of which a part is based on a *manifold* called *slow manifold*. To this end, a new approach based on certain aspects of the formalism of *Mechanics* and *Differential Geometry* was developed and led to a *geometrical* and *kinematics* interpretation of the evolution of the *trajectory curves*, integrals of these dynamical systems in the vicinity of the *slow manifold*, and allowed to study their stability.

Mechanics allowed, with the use of the velocity and instantaneous acceleration vectors, located on a point of the *trajectory curve*, to discriminate the *slow* domain from the *fast* domain and to locate the position of the *slow manifold* inside the phase space.

Certain notions of *Differential Geometry* like the expressions of *curvature*, *torsion* and that of the *osculating plane* provided an analytical equation of the *slow manifold* independent of the *slow eigenvectors* of the *tangent linear system*, therefore defined on a greater domain of the phase space.

The slow manifold was then considered as the location of the points where the *curvature* of the *trajectory curves*, integrals of these dynamical systems, is minimal (in dimension two this minimum becomes equal to zero). The sign of *torsion* allowed: to characterize its *attractivity*, to discriminate the *attractive* part from the *repulsive* part of the *slow manifold* and, to rule on the stability of these *trajectory curves.*

Thus, the presence in the phase space of an *attractive slow manifold* compelling the *trajectory curve*, integrals of the dynamic system to visit its vicinity allowed to analyse the *attractor* structure.

This approach based on certain aspects of the formalism of *Mechanics* and *Differential Geometry* and which was accompanied by the development of numerical programs made it possible to constitute a new tool for investigation of chaotic dynamical systems.

Its application to models of reference like that of B. Van der Pol., L.O. Chua or of E.N. Lorenz allowed obtaining more directly and with precision the analytical equation of their *slow manifold*. Moreover, a detailed study of the predator-*prey* models like that of Rosenzweig-MacArthur or Hastings-Powell, led on the one hand to the determination of their *slow manifold*and on the other hand to the design of a new three-dimensional model of *predator-prey* type: the Volterra-Gause model of which chaotic attractor has the shape of a snail shell (chaotic snail shell).

Keywords : *slow-fast dynamics; chaos; strange attractors; Frénet frame; curvature; torsion.*